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ON NEW INTERFACIAL FOUR-POTENTIAL ACOUSTIC SH-WAVE IN DISSIMILAR MEDIA PERTAINING TO TRANSVERSELY ISOTROPIC CLASS 6 *mm*

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ABSTRACT

This theoretical work documents one new interfacial shear-horizontal (SH) acoustic wave, the propagation of which is supported by the common interface between two dissimilar solid materials. For the treated case, four potentials (4P) such as the electric, magnetic, gravitational, and cogravitational potentials contribute in the wave motion in both dissimilar media pertaining to the transversely isotropic class 6 *mm*. This new interfacial acoustic SH-wave is guided by the perfectly bonded interface between two dissimilar solid continua. It was mathematically found the explicit form for calculation of the propagation velocity of the new interfacial wave. The existence condition for the new interfacial 4P-SH-wave was also discussed. Further developments in this research arena can be useful in constitution of various optical and microwave technical devices when some gravitational phenomena for new communication era technical devices must be taken into account. It is hoped that the contribution of some gravitational phenomena can be also helpful for the nondestructive testing and evaluation of common interfaces between two suitable solid materials as well as to record changes in the adhesive properties at the interface located within an adhesive bond joining two solids. The obtained theoretical results can be also used in educational purposes of undergraduate, graduate, and postgraduate students.

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INTRODUCTION

A homogeneous isotropic elastic bulk material (half-space) can release three types of bulk acoustic waves: one longitudinal (compressional) P-wave and two shear (incompressional or transversal) waves such as the shear-vertical (SV) and shear-horizontal (SH) waves. Concerning the mechanical displacements, the polarization vector of the bulk P-wave is parallel to the propagation direction and lies in the sagittal plane. The polarization vector of the bulk SV-wave is perpendicular to the propagation direction and lies in the sagittal plane. And the polarization vector of the bulk SH-wave is perpendicular to both the propagation direction and the sagittal plane. These all waves are purely mechanical waves and do not possess dispersion. In this isotropic case, only hybridization between two bulk P- and SV-waves can lead to the propagation of the Rayleigh surface acoustic wave (SAW) discovered by Rayleigh (1885).

The propagation velocity of this two-component Rayleigh SAW is slightly slower than the speed of the bulk SV-wave and the polarization vector remains in the sagittal plane. So, it is possible to state that the free surface of a bulk isotropic material, i.e. the interface between the solid and a vacuum can support the propagation of only one type of SAW because the bulk SH-wave remains lonely without a possibility of coupling with an additional component to form a second SAW, namely a nondispersive SH-SAW.

However, the simplest SH-SAW called the surface Love wave (Love, 1911) can be realized by the covering of the free surface of the bulk isotropic solid with a thin film (layer) of another isotropic solid. This is also two-component SAW because the SH-component of the layer is coupled with the SH-component of the bulk substrate (half-space). The surface Love wave represents the two-component dispersive SAW because the SH-SAW speed depends on the layer thickness and is confined between the SH bulk acoustic wave (SH-BAW) speeds in the layer

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and the substrate. This SAW has the following specific existence condition: the SH-BAW speed in the layer must be slower than the one in the substrate. For the case of thick layer that can be already treated as a second half-space above the bulk substrate, the Love wave speed approaches the SH-BAW speed for the layer. In this case, a coupling between the solid-solid interface and the solid-vacuum interface (i.e. free surface) is still present. This means that the propagation of purely interfacial acoustic SH-wave guided by the perfectly bonded interface between two isotropic solid half-spaces cannot be realized. Hence, the coupling of two components is not enough for any realization of propagation of nondispersive acoustic SH-wave along the interface of two dissimilar solids.

However, almost all solid materials represent anisotropic crystals. Some crystals can possess piezoelectric (PE) or piezomagnetic (PM), or both properties. The later crystals called piezoelectromagnetics (PEM) or magnetoelectroelastics (MEE) also possess the magnetoelectric (ME) effect and are classified as smart ME materials. The Rayleigh SAW and the second SAW such as SH-SAW can be realized in these bulk materials. In PE crystals there are high symmetry propagation directions (Lardat *et al*., 1971; Auld, 1990; Dieulesaint and Royer, 1980), in which the interface between the crystal and a vacuum can support the propagation of purely mechanical Rayleigh SAW and the second SAW called the surface Bleustein-Gulyaev (BG) wave discovered to the end of 1960s (Bleustein, 1968; Gulyaev, 1969). The surface BG wave can propagate on special cuts and in special propagation directions of the transversely isotropic materials of symmetry class 6 *mm*. The BG SAW has two components because the mechanical component is coupled with the electrical component. The problem of SAW propagation in anisotropic continua has been studied for many decades. It is of interest in acoustics of materials (for instance, nondestructive testing of materials) where the anisotropic elastic properties can be evaluated from measurements of the characteristics of laser-generated SAW (Chai and Wu, 1994; Hurley *et al*., 2001; Every, 2002; Every and Deschamps, 2003). It is of a great interest in seismology (Lee *et al*., 2008) and exploration geophysics (Tessmer and Kosloff, 1994).

For the case of the perfectly bonded interface between two dissimilar PE (6 *mm*) solid half-spaces, the nondispersive interfacial Maerfeld-Tournois wave (Maerfeld and Tournois, 1971) can propagate along their common interface. The propagation directions of the BG SH-SAW and the interfacial MT SH-wave are the same. The interfacial MT wave represents a four-component wave because there are the mechanical and electrical components in either solid. If one of two solids is nonpiezoelectric, the MT wave is three-component because the electrical component is absent in the nonpiezoelectric material. Therefore, the simplest interfacial wave is three-component. In the case of two similar PE materials, the speed of the interfacial MT SH-wave naturally reduces to the BG SH-SAW speed. Therefore, the interfacial BG SH-wave is used to study some plane defects in crystals that can frequently occur during the crystal growth. All written in the context above is also true when a PM material is used instead of a PE material.

The problem of wave propagation in the PEM material possessing the piezoelectric, piezomagnetic, and magnetoelectric effects is more complicated comparing with the PE or PM material. As a result, the PEM SH-SAW called the surface Bleustein-Gulyaev-Melkumyan (BGM) wave was discovered by Melkumyan (2007) only in four decades after the discovery by Bleustein (1968) and Gulyaev (1969). The surface BGM wave propagating in the transversely isotropic (Melkumyan, 2007; Zakharenko, 2013a; Zakharenko, 2010) and cubic (Zakharenko, 2011) PEM materials represents an analogy of the surface BG wave propagating in a PE or PM material. The PEM materials are more reach concerning the new acoustic SH-wave existence. It is expected that as many as twenty PEM SH-SAWs can exist in the transversely isotropic PEM materials, for instance, see in (Zakharenko, 2013a; Zakharenko, 2010; Zakharenko, 2013b; Zakharenko, 2015a; Zakharenko, 2015b), and only seven SH-SAWs (Zakharenko, 2011) were found in cubic PEM crystals. Regarding to a PE or PM material, only two SH-SAWs can propagate along the interface between the solid and a vacuum: the BG wave (Bleustein, 1968; Gulyaev, 1969) and the Bleustein wave (Bleustein, 1968) in the 6 *mm* materials; and the BG wave and the new SH-SAW (Zakharenko, 2007; Zakharenko, 2012a) in the cubic crystals. Maerfeld and Tournois (1971) have also discovered two different cases of the interfacial SH-waves. And there can be found thirty different interfacial SH-waves (Soh *et al*., 2006; Huang *et al*., 2009; Zakharenko, 2012b; Zakharenko, 2015c) in the 6 *mm* PEM materials.

The strong PE, PM, and PEM materials can be frequently fragile. This results in various imperfections of used surfaces and interfaces of manufactured (composite) structures due to adhesion and cohesion. Adhesion can frequently exist due to the presence of attractive forces operating at the interface between dissimilar contacting surfaces and cohesion can exist due to internal strengths in the same material (von Fraunhofer, 2012). Adhesive bonds are used extensively in the manufacture of (composite) structures that are particularly common in the aerospace industry. The nondestructive evaluation (NDE) in these structures can be utilized for classification of defects within these bonds: disbonds, weak interfacial bonding between the adhesive and one surface of the lap joint, cohesive weakness within the bond itself, and others. The acoustic SH waves can be produced more effortlessly by electromagnetic acoustic transducers (EMATs) in PEM materials in comparison with the PE materials. The EMATs offer a series of advantages over traditional piezoelectric transducers (Pei *et al*., 2012; Ribichini *et al*., 2010; Thompson, 1990; Hirao and Ogi, 2003).

It is well-known that an acoustic wave generated in a solid transmits energy from the wave generator to the wave detector. It was mentioned in (Füzfa, 2016) that gravitational forces coexist with any type of energy. Therefore, it is natural to additionally treat gravitational phenomena and the resulting system will consist of the mechanical, electrical, magnetic, gravitational, and cogravitational subsystems (Zakharenko, 2016; Zakharenko, 2017). This means that the propagation of the acoustic wave is coupled with the electrical, magnetic, gravitational, and cogravitational potentials, i.e. the case of the four-potential (4P) acoustic wave. In a PEM solid, the speed of an acoustic wave is five orders smaller than the speed of the electromagnetic wave. Therefore, the quasistatic approximation is used in the theory of acoustic wave propagation in PEM materials (Zakharenko, 2010; Zakharenko, 2011) when the wave propagation is coupled with both the electrical and magnetic potentials.

Heaviside (1893) has initiated the study of an analogy between the electromagnetism and the corresponding gravitational phenomena. And Jefimenko (2006) is the one of many researchers who has studied gravitation and cogravitation and therefore, contributed in the development of this analogy. As a result, the gravitational waves were detected in (Abbott *et al*., 2016) by a large group of thousand researchers. The existence of the gravitational waves was predicted by Einstein (1916). It is natural that the gravitational waves can interact with the electromagnetic waves (Kleidis *et al*., 2010; Forsberg *et al*., 2010). It a solid possessing the five subsystems mentioned above (i.e. the mechanical, electrical, magnetic, gravitational, and cogravitational subsystems) the interactions between some pairs of the subsystems were theoretically studied by Füzfa (2016) and (Li and Torr, 1991; Li and Torr, 1992; Torr and Li, 1993).

The following section releases the theoretical study of the interfacial acoustic SH-wave propagation along the common interface between two dissimilar solids. Both solids pertain to the transversely isotropic class 6 *mm*. And all five subsystems mentioned above are taken into account for either solid. In order to develop the theory of the interfacial acoustic SH-wave propagation coupled with the four potentials, the following section naturally uses already developed theory (Zakharenko, 2016; Zakharenko, 2017) concerning the 4P-SH-SAW propagation and compares the obtained results.

Theoretical findings

To describe a wave motion in a system consisting of the mechanical, electrical, magnetic, gravitational, and cogravitational subsystems is quite complicated (Zakharenko, 2016). To describe it, theory (Zakharenko, 2016) starts with the thermodynamics for an adiabatic process, obtains the coupled constitutive relations, and then applies the quasistatic approximation. As a result, the coupled equations of motion representing seven homogeneous partial differential equations of the second order can be obtained. These equations couple the elastic, electric (E), magnetic (H), gravitational (GE), and cogravitational (GH) fields representing the thermodynamic variables that can contribute in the wave motion. For the mechanical subsystem, the stress tensor σ_{ij} and the strain tensor τ_{ij} respectively represent the thermodynamic function and thermodynamic variable. The later has the following dependence on the mechanical displacements for small perturbations: $\tau_{ij} = 0.5(\partial U_i/\partial x_j + \partial U_j/\partial x_i)$, where x_j are the components of the real space and the indices i and j run from 1 to 3. The other thermodynamic functions are the electrical (D), magnetic (B), gravitational (GD), and cogravitational (GB) inductions. Therefore, three mechanical displacement components U_1, U_2, and U_3 can be coupled with the electrical (φ), magnetic (ψ), gravitational (Φ), and cogravitational (Ψ) potentials, where $E_i = -\partial\varphi/\partial x_i$, $H_i = -\partial\psi/\partial x_i$, $GE_i = -\partial\Phi/\partial x_i$, $GH_i = -\partial\Psi/\partial x_i$.

For this case of the SH-wave propagation in the transversely isotropic materials of class 6 *mm*, it is unnecessary to write down all seven coupled equations of motion in the differential form. Instead of that, it is necessary to treat the high symmetry propagation directions for the 6 *mm* material (Zakharenko, 2016; Lardat *et al*. 1971; Auld, 1990; Dieulesaint and Royer, 1980), in which the coupled equations of motion can be separated into two independent groups. The first group of two equations is for the propagation of the purely mechanical Rayleigh SAW that is out of this study. The second group of five equations of the acoustic SH-wave coupled with the electrical, magnetic, gravitational, and cogravitational potentials.

In any propagation direction on any crystal cut, the solutions of these seven coupled equations of motion can be introduced in the following plane wave form: $U_I = U_I^0 \exp[\mathrm{j}(k_1x_1 + k_2x_2 + k_3x_3 - \omega t)]$, where the index I runs from 1 to 7. $\mathrm{j} = (-1)^{1/2}$ is the imaginary unity and ω and t stand for angular frequency and time, respectively. $(k_1, k_2, k_3) = k(n_1, n_2, n_3)$ are the wavevector components and k is the wavenumber in the propagation direction because the directional cosines n_1, n_2, and n_3 are defined by $n_1 = 1$, $n_2 = 0$, and $n_3 \equiv n_3$. The values of U_1^0, U_2^0, U_3^0, $U_4^0 = \varphi^0$, $U_5^0 = \psi^0$, $U_6^0 = \Phi^0$, and $U_7^0 = \Psi^0$ are called the eigenvector components because there are $\left(U_1, U_2, U_3, U_4 = \varphi, U_5 = \psi, U_6 = \Phi, U_7 = \Psi\right)$.

Employing these plane wave solutions, the seven coupled equations of motion can be then rewritten in a compact tensor form known as the modified Green-Christoffel equation (Zakharenko, 2016) with the tensor components GL_{IJ}. This tensor form of seven homogeneous equations is generally written as follows:

$$\left(GL_{IJ} - \delta_{IJ}\rho V_{ph}\right)U_I^0 = 0 \tag{1}$$

In expression (1), the indices I and J run from 1 to 7. ρ is the mass density and the phase velocity is defined by $V_{ph} = \omega/k$. δ_{IJ} represents the Kronecker delta-function: $\delta_{IJ} = 1$ for $I = J < 4$, $\delta_{IJ} = 0$ for $I \neq J$, and $\delta_{44} = \delta_{55} = \delta_{66} = \delta_{77} = 0$. Compact form (1) represents the common problem for determination of the eigenvalues n_3 and the corresponding eigenvectors $\left(U_1^0, U_2^0, U_3^0, \varphi^0, \psi^0, \Phi^0, \Psi^0\right)$.

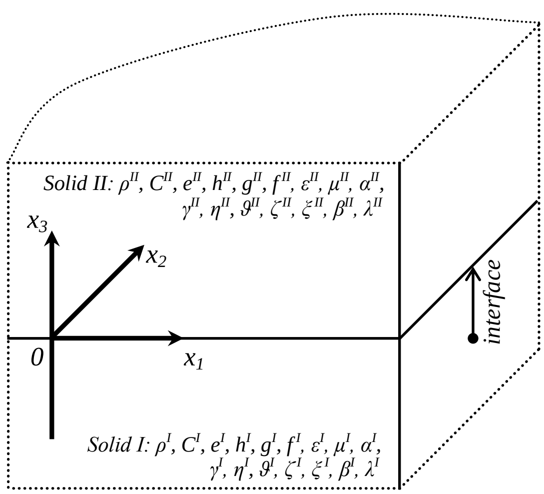

Fig. 1. The configuration of the structure consisting of two dissimilar solid half-spaces perfectly coupled at the common interface. The material parameters for both the solid continua are listed in table 1.

Consider the acoustic wave propagation in the high symmetry propagation directions of the transversely isotropic materials of class 6 *mm*. The configuration of two dissimilar solid continua perfectly coupled at their common interface is shown in figure 1 and all the material parameters present in the figure are listed in table 1. Review paper by Gulyaev (1998) states that the propagation direction in the 6 *mm* materials must be perpendicular to the 6-fold symmetry axis. Therefore, the rectangular coordinate system in figure 1 shows that both the x_1- and x_3-axes are directed perpendicular to the 6-fold symmetry axis. The later axis is managed along the x_2-axis. The coordinate beginning is situated at the interface. The acoustic wave propagates along the x_1-axis. So, the propagation direction is the same in both media. It is necessary to use the superscripts "I" and "II" for the first and second solids, respectively, in order to distinguish them. Therefore, expressions from (1) to (8) are valid for each solid separately when the corresponding superscript "I" or "II" is added to all the material parameters and wave characteristics. However, it is natural to use these superscripts in the boundary conditions and in the final expression because they already couple both dissimilar materials.

In the treated propagation direction, the purely SH-waves with the anti-plane polarization can propagate. They are also coupled with the electrical (φ), magnetic (ψ), gravitational (Φ), and cogravitational (Ψ) potentials. Therefore, equation (1) reduces to the following five homogeneous equations for the case of the acoustic SH-wave propagation having only single mechanical displacement component U_2:

$$\begin{pmatrix} C\left[m - \left(V_{ph}/V_{t4}\right)^2\right] & em & hm & gm & fm \\ em & -\varepsilon m & -\alpha m & -\zeta m & -\xi m \\ hm & -\alpha m & -\mu m & -\beta m & -\lambda m \\ gm & -\zeta m & -\beta m & -\gamma m & -\vartheta m \\ fm & -\xi m & -\lambda m & -\vartheta m & -\eta m \end{pmatrix} \begin{pmatrix} U^0 \\ \varphi^0 \\ \psi^0 \\ \Phi^0 \\ \Psi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{2}$$

In equation (2), $m = 1 + n_3^2$ and $V_{t4} = (C/\rho)^{1/2}$ is the purely mechanical shear-horizontal bulk acoustic wave (SH-BAW). All the material parameters are listed in table 1. All the suitable eigenvalues n_3 and the corresponding eigenvectors $\left(U^0, \varphi^0, \psi^0, \Phi^0, \Psi^0\right) = \left(U_2^0, U_4^0, U_5^0, U_6^0, U_7^0\right)$ must be found. In order to find the suitable eigenvalues n_3, the determinant of the coefficient matrix in equation (2) is equal to zero. Expanding this determinant of the (5 × 5) matrix, a polynomial of the tenth order is obtained. Therefore there are the following polynomial roots representing all possible values of the eigenvalues n_3:

$$n_3^{(1,2)} = n_3^{(3,4)} = n_3^{(5,6)} = n_3^{(7,8)} = \pm \mathrm{j} \tag{3}$$

$$n_3^{(9,10)} = \pm \mathrm{j}\sqrt{1 - \left(V_{ph}/V_{temgc}\right)^2} \tag{4}$$

where

$$V_{temgc} = \sqrt{C/\rho}\left(1 + K_{emgc}^2\right)^{1/2} \tag{5}$$

$$K_{emgc}^2 = \frac{A_1}{A_2} \quad (6)$$

$$\begin{aligned} A_1 &= e^2\left(\mu\gamma\eta + 2\beta\lambda\vartheta - \lambda^2\gamma - \beta^2\eta - \vartheta^2\mu\right) \\ &+ h^2\left(\varepsilon\gamma\eta + 2\zeta\xi\vartheta - \vartheta^2\varepsilon - \zeta^2\eta - \xi^2\gamma\right) \\ &+ g^2\left(\varepsilon\mu\eta + 2\alpha\xi\lambda - \lambda^2\varepsilon - \alpha^2\eta - \xi^2\mu\right) \\ &+ f^2\left(\varepsilon\mu\gamma + 2\alpha\beta\zeta - \beta^2\varepsilon - \alpha^2\gamma - \zeta^2\mu\right) \\ &+ 2eh\left(\vartheta^2\alpha + \zeta\beta\eta + \xi\gamma\lambda - \alpha\gamma\eta - \zeta\lambda\vartheta - \xi\beta\vartheta\right) \\ &+ 2eg\left(\alpha\beta\eta + \lambda^2\zeta + \xi\vartheta\mu - \alpha\lambda\vartheta - \zeta\mu\eta - \xi\beta\lambda\right) \\ &+ 2ef\left(\alpha\gamma\lambda + \zeta\vartheta\mu + \beta^2\xi - \alpha\beta\vartheta - \zeta\beta\lambda - \xi\mu\gamma\right) \\ &+ 2hg\left(\varepsilon\lambda\vartheta + \zeta\alpha\eta + \xi^2\beta - \varepsilon\eta\beta - \zeta\lambda\xi - \xi\vartheta\alpha\right) \\ &+ 2hf\left(\varepsilon\beta\vartheta + \zeta^2\lambda + \xi\alpha\gamma - \varepsilon\lambda\gamma - \zeta\vartheta\alpha - \xi\zeta\beta\right) \\ &+ 2gf\left(\varepsilon\beta\lambda + \alpha^2\vartheta + \xi\mu\zeta - \varepsilon\mu\vartheta - \alpha\zeta\lambda - \alpha\beta\xi\right) \end{aligned} \quad (7)$$

$$\begin{aligned} A_2 &= C\left(\varepsilon\mu - \alpha^2\right)\left(\gamma\eta - \vartheta^2\right) \\ &+ C\left(\beta^2\xi^2 - \xi^2\mu\gamma - \beta^2\varepsilon\eta\right) + C\left(\lambda^2\zeta^2 - \lambda^2\varepsilon\gamma - \zeta^2\mu\eta\right) \\ &+ 2C\left(\gamma\alpha\xi\lambda + \eta\alpha\beta\zeta + \varepsilon\beta\lambda\vartheta + \mu\zeta\xi\vartheta - \zeta\xi\beta\lambda - \alpha\zeta\lambda\vartheta - \alpha\beta\xi\vartheta\right) \end{aligned} \quad (8)$$

In definition (5), V_{temgc} is the speed of the SH-BAW coupled with the electrical, magnetic, gravitational, and cogravitational potentials. In definition (6), K_{emgc}^2 is called the coefficient of the electromagnetogravito-cogravitomechanical coupling (CEMGCMC).

The found ten eigenvalues (3) and (4) can be used in equation (2) for finding of the suitable eigenvector components $\left(U^0, \varphi^0, \psi^0, \Phi^0, \Psi^0\right)$. For both solids, it is possible for simplicity to exploit with the corresponding superscript "I" or "II" already obtained eigenvectors (33) and (34) from (Zakharenko, 2017). Indeed, to obtain the eigenvectors represents a quite complicated mathematical problem and it is unnecessary here to rewrite many complicated and very large expressions.

This is the case of the interfacial acoustic SH-wave propagating along the x_1-axis as it is shown in figure 1. This case has a peculiarity concerning the choice of the obtained eigenvalues n_3. The wave must damp from the common interface towards the depth of either solid. To satisfy this damping condition, only five eigenvalues n_3 possessing a negative (positive) sign must be further used for solid I (solid II). With the chosen eigenvalues and their corresponding eigenvectors, it is possible to construct the following complete parameters: the complete electrical potential (φ^Σ), complete magnetic potential (ψ^Σ), complete gravitational potential (Φ^Σ), and complete cogravitational potential (Ψ^Σ). These complete parameters for the first and second solids are respectively composed as follows:

Table 1. The material parameters for both the solid media where the superscripts "I" and "II" are for the first and second dissimilar solids.

Material parameter	Symbol	Solid I	Solid II	Fundamental dimension
Mass density	ρ	ρ^{I}	ρ^{II}	kg/m^3
Elastic stiffness constant	$C = C_{44} = C_{66}$	C^{I}	C^{II}	kg/(m×s^2)
Piezoelectric constant	$e = e_{16} = e_{34}$	e^{I}	e^{II}	kg$^{1/2}$/m$^{3/2}$
Piezomagnetic coefficient	$h = h_{16} = h_{34}$	h^{I}	h^{II}	kg$^{1/2}$/(m$^{1/2}$×s)
Piezogravitic constant	$g = g_{16} = g_{34}$	g^{I}	g^{II}	kg/m^2
Piezocogravitic coefficient	$f = f_{16} = f_{34}$	f^{I}	f^{II}	s^{-1}
Dielectric permittivity coefficient (electric constant)	$\varepsilon = \varepsilon_{11} = \varepsilon_{33}$	ε^{I}	$\varepsilon^{\mathrm{II}}$	s^2/m^2
Magnetic permeability coefficient (magnetic constant)	$\mu = \mu_{11} = \mu_{33}$	μ^{I}	μ^{II}	dimensionless
Electromagnetic constant	$\alpha = \alpha_{11} = \alpha_{33}$	α^{I}	α^{II}	s/m
Gravitic constant (gravitoelectric permittivity coefficient)	$\gamma = \gamma_{11} = \gamma_{33}$	γ^{I}	γ^{II}	kg×s^2/m^3
Cogravitic constant (gravitomagnetic permeability coefficient)	$\eta = \eta_{11} = \eta_{33}$	η^{I}	η^{II}	m/kg
Gravitocogravitic constant	$\vartheta = \vartheta_{11} = \vartheta_{33}$	ϑ^{I}	ϑ^{II}	s/m
Gravitoelectric constant (electrogravitic constant)	$\zeta = \zeta_{11} = \zeta_{33}$	ζ^{I}	ζ^{II}	kg$^{1/2}$×s^2/m$^{5/2}$
Cogravitoelectric constant (electrocogravitic constant)	$\xi = \xi_{11} = \xi_{33}$	ξ^{I}	ξ^{II}	s/(kg$^{1/2}$×m$^{1/2}$)
Gravitomagnetic constant (magnetogravitic constant)	$\beta = \beta_{11} = \beta_{33}$	β^{I}	β^{II}	kg$^{1/2}$×s/m$^{3/2}$
Cogravitomagnetic constant (magnetocogravitic constant)	$\lambda = \lambda_{11} = \lambda_{33}$	λ^{I}	λ^{II}	m$^{1/2}$/kg$^{1/2}$

$$U_J^{\Sigma\mathrm{I}} = \sum_{s=1,3,5,7,9} F^{\mathrm{I}(s)} U_J^{0\mathrm{I}(s)} \exp\left[\mathrm{j}\left(k_1x_1 + k_2x_2 + k_3^{\mathrm{I}(s)}x_3 - \omega t\right)\right]$$
$$= F^{\mathrm{I}} U_J^{0\mathrm{I}(7)} \exp\left[\mathrm{j}k\left(x_1 + n_3^{\mathrm{I}(7)}x_3 - V_{ph}t\right)\right]$$
$$+ F^{\mathrm{I2}} U_J^{0\mathrm{I}(9)} \exp\left[\mathrm{j}k\left(x_1 + n_3^{\mathrm{I}(9)}x_3 - V_{ph}t\right)\right] \quad (9)$$

$$U_J^{\Sigma\mathrm{II}} = \sum_{s=2,4,6,8,10} F^{\mathrm{II}(s)} U_J^{0\mathrm{II}(s)} \exp\left[\mathrm{j}\left(k_1x_1 + k_2x_2 + k_3^{\mathrm{II}(s)}x_3 - \omega t\right)\right]$$
$$= F^{\mathrm{II}} U_J^{0\mathrm{II}(8)} \exp\left[\mathrm{j}k\left(x_1 + n_3^{\mathrm{II}(8)}x_3 - V_{ph}t\right)\right]$$
$$+ F^{\mathrm{II2}} U_J^{0\mathrm{II}(10)} \exp\left[\mathrm{j}k\left(x_1 + n_3^{\mathrm{II}(10)}x_3 - V_{ph}t\right)\right] \quad (10)$$

where the index J = 2, 4, 5, 6, 7.

In expression (9), $F^{\mathrm{I}} = F^{\mathrm{I}(1)} + F^{\mathrm{I}(3)} + F^{\mathrm{I}(5)} + F^{\mathrm{I}(7)}$ and $F^{\mathrm{I2}} = F^{\mathrm{I}(9)}$. In expression (10), $F^{\mathrm{II}} = F^{\mathrm{II}(2)} + F^{\mathrm{II}(4)} + F^{\mathrm{II}(6)} + F^{\mathrm{II}(8)}$ and $F^{\mathrm{II2}} = F^{\mathrm{II}(10)}$. In expressions (9) and (10), it is natural to use the index s = 1, 3, 5, 7, 9 and s = 2, 4, 6, 8, 10 for the first and second solids, respectively. It is indeed possible to use the index s = 1, 3, 5, 7, 9 for the second solid, too. In this case, one has however to change the sign of each eigenvalues n_3.

These complete parameters (9) and (10) are next used for construction of the (10 × 10) boundary conditions' determinant (BCD). To construct this determinant there must be written ten homogeneous equations in ten unknowns $F^{\mathrm{I}(1)}$, $F^{\mathrm{I}(3)}$, $F^{\mathrm{I}(5)}$, $F^{\mathrm{I}(7)}$, $F^{\mathrm{I}(9)}$, $F^{\mathrm{II}(2)}$, $F^{\mathrm{II}(4)}$, $F^{\mathrm{II}(6)}$, $F^{\mathrm{II}(8)}$, $F^{\mathrm{II}(10)}$. Ten homogeneous equations must correspond to ten boundary conditions. Therefore, a pair of boundary conditions for each subsystem must be applied at the perfectly bonded common interface between two dissimilar solids. There are the mechanical, electrical, magnetic, gravitational, and cogravitational subsystems.

At the interface x_3 = 0 in figure 1, the mechanical boundary conditions include the continuity of both the mechanical displacement and the normal component of the stress tensor: $U^{\mathrm{I}} = U^{\mathrm{II}}$ and $(\sigma_{32})^{\mathrm{I}} = (\sigma_{32})^{\mathrm{II}}$. The electrical, magnetic, gravitational, and cogravitational boundary conditions at x_3 = 0 are: $\varphi^{\mathrm{I}} = 0$, $\varphi^{\mathrm{II}} = 0$, $\psi^{\mathrm{I}} = 0$, $\psi^{\mathrm{II}} = 0$, $\Phi^{\mathrm{I}} = 0$, $\Phi^{\mathrm{II}} = 0$, $\Psi^{\mathrm{I}} = 0$, $\Psi^{\mathrm{II}} = 0$. Using for simplicity the weight factors F^{I}, F^{I2}, F^{II}, F^{II2} defined after expression (10), ten homogeneous equations corresponding to the ten boundary conditions stated above are respectively inscribed as follows:

$$F^{\mathrm{I2}} U^{0\mathrm{I}(9)} - F^{\mathrm{II2}} U^{0\mathrm{II}(10)} = 0 \quad (11)$$

$$F^{\mathrm{I}}\left[e^{\mathrm{I}}\varphi^{0\mathrm{I}(7)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(7)} + g^{\mathrm{I}}\Phi^{0\mathrm{I}(7)} + f^{\mathrm{I}}\Psi^{0\mathrm{I}(7)}\right]$$
$$+ F^{\mathrm{I2}} b^{\mathrm{I}}\left[C^{\mathrm{I}}U^{0\mathrm{I}(9)} + e^{\mathrm{I}}\varphi^{0\mathrm{I}(9)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(9)} + g^{\mathrm{I}}\Phi^{0\mathrm{I}(9)} + f^{\mathrm{I}}\Psi^{0\mathrm{I}(9)}\right]$$
$$+ F^{\mathrm{II}}\left[e^{\mathrm{II}}\varphi^{0\mathrm{II}(8)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(8)} + g^{\mathrm{II}}\Phi^{0\mathrm{II}(8)} + f^{\mathrm{II}}\Psi^{0\mathrm{II}(8)}\right]$$
$$+ F^{\mathrm{II2}} b^{\mathrm{II}}\left[C^{\mathrm{II}}U^{0\mathrm{II}(10)} + e^{\mathrm{II}}\varphi^{0\mathrm{II}(10)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(10)} + g^{\mathrm{II}}\Phi^{0\mathrm{II}(10)} + f^{\mathrm{II}}\Psi^{0\mathrm{II}(10)}\right] = 0 \quad (12)$$

$$F^{\mathrm{I}}\varphi^{0\mathrm{I}(7)} + F^{\mathrm{I2}}\varphi^{0\mathrm{I}(9)} = 0 \quad (13)$$

$$F^{\mathrm{II}}\varphi^{0\mathrm{II}(8)} + F^{\mathrm{II2}}\varphi^{0\mathrm{II}(10)} = 0 \quad (14)$$

$$F^{\mathrm{I}}\psi^{0\mathrm{I}(7)} + F^{\mathrm{I2}}\psi^{0\mathrm{I}(9)} = 0 \quad (15)$$

$$F^{\mathrm{II}}\psi^{0\mathrm{II}(8)} + F^{\mathrm{II2}}\psi^{0\mathrm{II}(10)} = 0 \quad (16)$$

$$F^{\mathrm{I}}\Phi^{0\mathrm{I}(7)} + F^{\mathrm{I2}}\Phi^{0\mathrm{I}(9)} = 0 \quad (17)$$

$$F^{\mathrm{II}}\Phi^{0\mathrm{II}(8)} + F^{\mathrm{II2}}\Phi^{0\mathrm{II}(10)} = 0 \quad (18)$$

$$F^{\mathrm{I}}\Psi^{0\mathrm{I}(7)} + F^{\mathrm{I2}}\Psi^{0\mathrm{I}(9)} = 0 \quad (19)$$

$$F^{\mathrm{II}}\Psi^{0\mathrm{II}(8)} + F^{\mathrm{II2}}\Psi^{0\mathrm{II}(10)} = 0 \quad (20)$$

where

$$b^{\mathrm{I}} = \sqrt{1 - \left(V_{ph}/V_{temgc}^{\mathrm{I}}\right)^2} \quad (21)$$

$$b^{\mathrm{II}} = \sqrt{1 - \left(V_{ph}/V_{temgc}^{\mathrm{II}}\right)^2} \quad (22)$$

Ten homogeneous equations from (11) to (20) can be written in a matrix form and then the corresponding (10 × 10) determinant of the coefficient matrix can be composed. It is natural to treat equation (12) as a linear combination of equations (13), (14), (15), (16), (17), (18), (19), (20). For this purpose, it is necessary to respectively multiply these eight equations by the following factors: e^{I}, e^{II}, h^{I}, h^{II}, g^{I}, g^{II}, f^{I}, f^{II}. It is well-known that a determinant is equal to zero when a row (column) of the determinant represents a linear combination of two or several rows (columns) of the determinant. It is natural to treat rows of the (10 × 10) determinant. Therefore, it is necessary to use equation (11) and to successively subtract equations (13), (14), (15), (16), (17), (18), (19), (20) with the corresponding factors e^{I}, e^{II}, h^{I}, h^{II}, g^{I}, g^{II}, f^{I}, f^{II} from equation (12). The resulting equation obtained from equitation (12) can be further transformed with equations (A1) and (A2) from Appendix I of paper by Zakharenko (2016). As a result, it is possible to obtain a simplified equation for calculation of the propagation velocity of the interfacial acoustic 4P-SH-wave.

Therefore, the velocity V_{new_in} of the new interfacial wave can be evaluated with the following formula:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\sqrt{1 - \left(V_{new\_in}/V_{temgc}^{\mathrm{I}}\right)^2}\left[1 + \left(K_{emgc}^2\right)^{\mathrm{I}}\right]$$
$$+ \sqrt{1 - \left(V_{new\_in}/V_{temgc}^{\mathrm{II}}\right)^2}\left[1 + \left(K_{emgc}^2\right)^{\mathrm{II}}\right] \quad (23)$$
$$= \frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\left(K_{emgc}^2\right)^{\mathrm{I}} + \left(K_{emgc}^2\right)^{\mathrm{II}}$$

In order to have a solution of equation (23), it is preferable to deal with the following conditions: $C^{\mathrm{I}} \sim C^{\mathrm{II}}$ and $\left(K_{emgc}^2\right)^{\mathrm{I}} \sim \left(K_{emgc}^2\right)^{\mathrm{II}}$ resulting in $V_{temgc}^{\mathrm{I}} \sim V_{temgc}^{\mathrm{II}}$. Also, there must be $V_{new\_in} < V_{temgc}^{\mathrm{I}}$ and $V_{new\_in} < V_{temgc}^{\mathrm{II}}$. In general, the nondimensional coefficients $\left(K_{emgc}^2\right)^{\mathrm{I}} < 1$ and $\left(K_{emgc}^2\right)^{\mathrm{II}} < 1$, even frequently $\left(K_{emgc}^2\right)^{\mathrm{I}} << 1$ and $\left(K_{emgc}^2\right)^{\mathrm{II}} << 1$. As a result, the speed V_{new_in} of the new interfacial acoustic 4P-SH-wave must be generally slightly slower than the slowest speed of the 4P-SH-BAWs V_{temgc}^{I} and V_{temgc}^{II}. This is the main recommendation concerning the choice of dissimilar materials. In the case of two similar materials, equation (23) reduces to the new 4P-SH-SAW defined by formula (78) from (Zakharenko, 2016).

CONCLUSION

This theoretical work has demonstrated that an interfacial acoustic 4P-SH-wave can propagate along the common interface between two solid half-spaces. The case of the perfectly bonded interface between two dissimilar solids belonging to the transversely isotropic materials of class 6 *mm* was considered. The formula for calculation of the speed of the new interfacial SH-wave was obtained in an explicit form and some recommendations on choice of suitable materials to realize the propagation of the new interfacial wave is given. The obtained theoretical results can be used in educational purposes of undergraduate, graduate, and postgraduate students. However, interfacial acoustic waves are also used in the nondestructive testing and evaluation of common interfaces between two solids. Also, the new nondispersive interfacial wave coupled with the four potential is a good candidate for constitution of various technical devices for new communication era based on some gravitational phenomena.

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